Electromagnetic Meson Form Factor from a Relativistic Coupled-Channel Approach

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Abstract

Point-form relativistic quantum mechanics is used to derive an expression for the electromagnetic form factor of a pseudoscalar meson for space-like momentum transfers. The elastic scattering of an electron by a confined quark-antiquark pair is treated as a relativistic two-channel problem for the $q\bar{q}e$ and $q\bar{q}e\gamma$ states. With the approximation that the total velocity of the $q\bar{q}e$ system is conserved at (electromagnetic) interaction vertices this simplifies to an eigenvalue problem for a Bakamjian-Thomas type mass operator. After elimination of the $q\bar{q}e\gamma$ channel the electromagnetic meson current and form factor can be directly read off from the one-photon-exchange optical potential. By choosing the invariant mass of the electron-meson system large enough, cluster separability violations become negligible. An equivalence with the usual front-form expression, resulting from a spectator current in the $q^+ = 0$ reference frame, is established. The generalization of this multichannel approach to electroweak form factors for an arbitrary bound few-body system is quite obvious. By an appropriate extension of the Hilbert space this approach is also able to accommodate exchange-current effects.

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I. INTRODUCTION

Elastic electron-hadron scattering is an important source of information on the internal structure of hadrons. Usually it is treated in the one-photon-exchange approximation so that the invariant scattering amplitude can be written as the contraction of a (point-like) electron current with a hadron current times the photon propagator [35]. For the phenomenological analysis of electron-hadron scattering the most general structure of the hadron current, compatible with Poincaré covariance and current conservation, is assumed. The hadron current is a sum of independent 4-vectors multiplied by Lorentz invariant functions, the hadron form factors. The form factors are the observables which encode the electromagnetic structure of the hadron. The kinematic quantities that are available for constructing the hadron current are the 4-momenta of the incoming and outgoing hadron. The form factors are thus functions of the only Lorentz invariant variable one can build from these 4-vectors, namely the 4-momentum-transfer squared.

The theoretical analysis of hadron form factors amounts to asking how the electromagnetic current of the hadron may be expressed in terms of the electromagnetic currents of the constituents. The fact that the hadron current cannot be a simple sum of the constituent currents has long been recognized [1]. From the transformation properties of the current operator $\hat{J}_\mu(x)$ under Poincaré transformations it is quite obvious that the transformed operator $\hat{J}'_\mu(x')$ will, in general, be interaction dependent as soon as some of the Poincaré generators contain interaction terms. The binding forces must thus also show up in the hadron current. How the binding interaction enters the hadron current is further restricted by current conservation and the physical condition that there be no renormalization of the hadron charge. The latter means that the hadron charge should be equal to the sum of the constituent charges, independent on whether the binding forces are present or not. For relativistic quantum mechanics of systems with a fixed number of particles the most general form of Poincaré covariant tensor operators in a given dynamical model has been derived in Ref. [2]. One of the conclusions of the authors is that relativity alone does not provide any strong dynamical constraints on the electromagnetic current operator. A whole class of current operators for two- and three-particle bound states, which satisfies the requirements of Poincaré covariance, current conservation and cluster separability, has been formally constructed in Ref. [3], where the existence of solutions for an arbitrary number of

particles has also been proved. For a comprehensive survey on the problem of constructing the electromagnetic current operator of a bound few-body system within the framework of relativistic quantum mechanics and the different attempts to solve this problem we refer to the introduction of Ref. [3].

The procedure for constructing current operators proposed in Ref. [3] is based on the point-form of relativistic quantum mechanics. This form is characterized by the property that all 4 generators of space-time translations are interaction dependent, whereas the generators of Lorentz transformations stay free of interactions. It thus provides a natural starting point for the construction of Lorentz-covariant tensor operators. Another issue which is naturally addressed in the point form is the problem of cluster separability, i.e. roughly speaking the property that arbitrary subsystems of an interacting system should not interact with each other, if they are separated by large space-like distances [4, 5, 6]. The cluster separability condition for the electromagnetic current operator means that it must become the sum of subsystem current operators if the interaction between the subsystems is turned off. Cluster separability is closely related to the previously mentioned requirement that the charge of the whole system should not be renormalized by the interaction.

By making use of the equivalence of the different forms of relativistic quantum mechanics the point-form results have also been obtained in the instant and front forms. It should be noted however, that all the results in Ref. [3] are rather formal and no explicit application is presented. Subsequently the whole form factor analysis has been redone in the front form of relativistic quantum mechanics [7] and applied to the calculation of electromagnetic properties of the deuteron [8] and the pion [9].

In the present paper we will also exploit the virtues of the point form of relativistic quantum mechanics for the calculation of meson form factors within constituent quark models. Our strategy, however, differs from the one in Ref. [3]. Rather than starting with the most general form of the electromagnetic current operator and trying to satisfy the covariance, current-conservation and cluster-separability constraints such that compatibility with a particular interaction model is achieved, we start with a Poincaré invariant treatment of electron-hadron scattering in which the dynamics of the exchanged photon is explicitly taken into account. Since the one-photon-exchange optical potential is expected to have the structure of a current-current interaction it should be possible to extract the hadron current. Once this is done it will be necessary to investigate whether the resulting current has all the

desired properties and if not whether and how its deficiencies can be cured.

The general Poincaré invariant framework which we use to describe electron-hadron scattering will be presented in Sec. II. It is a relativistic two-channel formalism for a Bakamjian-Thomas type mass operator [4, 10] in a velocity-state representation [11]. The two channels are either the electron-meson and electron-meson photon channels or the electron-quark-antiquark and electron-quark-antiquark-photon channels, depending on whether the problem is considered on the hadronic or constituent level. The field theoretical vertices for the emission and absorption of a photon are implemented in the Bakamjian-Thomas type framework as suggested in Ref. [12]. In Sec. II A we will first demonstrate on the hadronic level that the one-photon-exchange optical potential has indeed the structure of a current-current interaction with a conserved meson current that transforms like a 4-vector under Lorentz transformations. This holds even if one allows for a phenomenological vertex form factor at the photon-meson vertex. The whole calculation will be repeated on the constituent level with an instantaneous confining potential between quark and antiquark in Sec. II B. In this way a microscopic meson current which is conserved and behaves like a 4-vector can again be extracted from the one-photon-exchange optical potential. By comparing appropriate matrix elements of the one-photon-exchange optical potentials on the hadronic and on the constituent level the electromagnetic form factor will be identified in Sec. III. But it will turn out not to satisfy the required cluster properties. By going to the infinite momentum frame of the meson the form factor becomes a simple analytical expression and satisfies the cluster separability property. As a first numerical application the electromagnetic pion form factor will be calculated with a simple harmonic-oscillator wave function. In Sec. IV A it will be shown that our form factor result is equivalent to the usual front-form expression, resulting from a spectator current in the $q^+ = 0$ frame. Section IV B is devoted to a comparison with the so called "point-form spectator model" [13, 14] – which is another procedure for calculating electromagnetic hadron form factors in point-form quantum mechanics. A summary and the possibility for further generalizations and applications of the formalism presented in this paper will be given in Sec. V.

II. MASS OPERATOR FOR ELECTRON-MESON SCATTERING

As already mentioned in the introduction, we will treat the electromagnetic scattering of an electron by a pseudoscalar meson as a two-channel problem for an appropriately defined mass operator. This mass operator acts on a Hilbert space which is the direct sum of the incoming and outgoing electron-meson states and the intermediate electron-meson-photon states. Decomposing an arbitrary mass eigenstate of the system $|\psi\rangle$ into components belonging to these two channels, $|\psi\rangle = |\psi_{eM}\rangle + |\psi_{eM\gamma}\rangle$, the eigenvalue equation for the mass operator $\hat{M}|\psi\rangle = m|\psi\rangle$ becomes a system of coupled equations for the respective components:

$$\begin{aligned} \hat{M}_{eM}|\psi_{eM}\rangle + \hat{K}^\dagger|\psi_{eM\gamma}\rangle &= m|\psi_{eM}\rangle \,, \\ \hat{K}|\psi_{eM}\rangle + \hat{M}_{eM\gamma}|\psi_{eM\gamma}\rangle &= m|\psi_{eM\gamma}\rangle \,. \end{aligned} \tag{1}$$

$\hat{K}$ and $\hat{K}^\dagger$ are vertex operators that are responsible for the emission and absorption of a photon, $\hat{M}_{eM}$ and $\hat{M}_{eM\gamma}$ are mass operators for the free electron-meson and electron-meson-gamma systems, respectively. We can solve the second equation for $|\psi_{eM\gamma}\rangle$ and insert the resulting expression into the first one to end up with a non-linear eigenvalue equation for $|\psi_{eM}\rangle$:

$$\left(\hat{M}_{eM} - m\right)|\psi_{eM}\rangle = \hat{K}^\dagger\left(\hat{M}_{eM\gamma} - m\right)^{-1}\hat{K}|\psi_{eM}\rangle =: \hat{V}_{\rm opt}(m)\,|\psi_{eM}\rangle \,. \tag{2}$$

The right-hand side of this equation describes the action of the one-photon-exchange optical potential $\hat{V}_{\rm opt}$ on the $|\psi_{eM}\rangle$ state.

Particular matrix elements of the optical potential are needed for the identification and extraction of the electromagnetic meson form factor. We derive them first for the case where the composite nature of the meson is taken into account by means of a phenomenological form factor at the photon-meson vertex. In the sequel we will redo the calculation for the case where the meson is treated as a quark-antiquark bound state that has the quantum numbers of the meson. In the latter case the photon couples directly to the point-like constituent (anti)quark. A comparison of both cases will finally allow us to identify the electromagnetic current of the meson and to express the electromagnetic meson form factor in terms of the quark-antiquark bound-state wave function.

A. Hadronic level

Relativistic invariance of our quantum mechanical description of electron-meson scattering is guaranteed if one is able to find a realization of the Poincaré algebra in terms of operators that act on the direct sum of the eM and $eM\gamma$ Hilbert spaces. This can be achieved by means of the, so called, "Bakamjian-Thomas construction" [10]. Its point-form version amounts to the assumption that the free 4-velocity operator $\hat{V}^{\mu}_{\text{free}}$ can be factored out of the (interacting) 4-momentum operator,

$$\hat{P}^{\mu} = \hat{P}^{\mu}_{\text{free}} + \hat{P}^{\mu}_{\text{int}} = (\hat{M}_{\text{free}} + \hat{M}_{\text{int}})\, \hat{V}^{\mu}_{\text{free}}\,, \tag{3}$$

so that one can concentrate on studying the mass operator $\hat{M} = \hat{M}_{\text{free}} + \hat{M}_{\text{int}}$. The Poincaré algebra is satisfied provided that the interacting part of the mass operator $\hat{M}_{\text{int}}$ is a Lorentz scalar and commutes with $\hat{V}^{\mu}_{\text{free}}$. For this kind of construction it is advantageous to represent the mass operator and the Poincaré generators in a basis that consists of velocity states [11]. An n-particle velocity state $|v; \vec{k}_1, \mu_1; \vec{k}_2, \mu_2; \ldots; \vec{k}_n, \mu_n\rangle$ is simply obtained by starting from a multiparticle momentum state in its rest frame and boosting it to overall 4-velocity v ($v_\mu v^\mu = 1$) by means of a canonical spin boost $B_c(v)$ [4], i.e.

$$|v; \vec{k}_1, \mu_1; \vec{k}_2, \mu_2; \ldots; \vec{k}_n, \mu_n\rangle = \hat{U}_{B_c(v)}\, |\vec{k}_1, \mu_1; \vec{k}_2, \mu_2; \ldots; \vec{k}_n, \mu_n\rangle \quad \text{with} \quad \sum_{i=1}^{n} \vec{k}_i = 0\,. \tag{4}$$

The μ_is denote the spin projections of the individual particles. By construction one of the $\vec{k}_i$s is redundant. In the following we will make extensive use of orthogonality and completeness relations of the velocity states. For a system of $n \geq 2$ particles with 4-momenta k_i, masses m_i, energies $\omega_{k_i} := \sqrt{m_i^2 + \vec{k}_i^2}$, and spin projections μ_i, $i = 1, \ldots, n$, we have the orthogonality relation

$$\begin{aligned}
&\langle v'; \vec{k}'_1, \mu'_1; \vec{k}'_2, \mu'_2; \ldots; \vec{k}'_n, \mu'_n |\, v; \vec{k}_1, \mu_1; \vec{k}_2, \mu_2; \ldots; \vec{k}_n, \mu_n\rangle \\
&= v_0\, \delta^3(\vec{v}\,' - \vec{v})\, \frac{(2\pi)^3 2\omega_{k_n}}{\left(\sum_{i=1}^{n} \omega_{k_i}\right)^3} \left(\prod_{i=1}^{n-1} (2\pi)^3 2\omega_{k_i} \delta^3(\vec{k}'_i - \vec{k}_i) \right) \left(\prod_{i=1}^{n} \delta_{\mu'_i \mu_i} \right) .
\end{aligned} \tag{5}$$

Here we have taken (without loss of generality) the nth momentum to be redundant. The corresponding completeness relation reads:

$$\begin{aligned}
\mathbb{1}_{1,2,\ldots,n} = & \sum_{\mu_1=-j_1}^{j_1} \sum_{\mu_2=-j_2}^{j_2} \cdots \sum_{\mu_n=-j_n}^{j_n} \int \frac{d^3 v}{(2\pi)^3 v_0} \left(\prod_{i=1}^{n-1} \frac{d^3 k_i}{(2\pi)^3 2\omega_{k_i}} \right) \frac{\left(\sum_{i=1}^{n} \omega_{k_i}\right)^3}{2\omega_{k_n}} \\
& \times |v; \vec{k}_1, \mu_1; \vec{k}_2, \mu_2; \ldots; \vec{k}_n, \mu_n\rangle \langle v; \vec{k}_1, \mu_1; \vec{k}_2, \mu_2; \ldots; \vec{k}_n, \mu_n|\,,
\end{aligned} \tag{6}$$

where j_i is the spin of the ith particle. One of the big advantages of velocity states as compared with the usual momentum states is that under a Lorentz transformation Λ the Wigner rotation is the same for all particles, i.e.

$$\hat{U}_\Lambda|v;\vec{k}_1,\mu_1;\vec{k}_2,\mu_2;\ldots;\vec{k}_n,\mu_n\rangle = \sum_{\mu_1',\mu_2',\ldots,\mu_n'}\left(\prod_{i=1}^{n} D^{j_i}_{\mu_i'\mu_i}(R_{\mathrm{W}}(v,\Lambda))\right)$$
$$\times|\Lambda v;\overrightarrow{R_{\mathrm{W}}(v,\Lambda)k}_1,\mu_1';\overrightarrow{R_{\mathrm{W}}(v,\Lambda)k}_2,\mu_2';\ldots;\overrightarrow{R_{\mathrm{W}}(v,\Lambda)k}_n,\mu_n'\rangle\,, \tag{7}$$

with the Wigner-rotation matrix

$$R_{\mathrm{W}}(v,\Lambda) = B_c^{-1}(\Lambda v)\Lambda B_c(v)\,. \tag{8}$$

In a velocity-state basis the Bakamjian-Thomas type 4-momentum operator, Eq. (3), becomes diagonal in the 4-velocity v. This is a special feature of the Bakamjian-Thomas construction which, in general, does not hold for arbitrary interacting relativistic quantum theories. It is, in particular, not possible to factorize the 4-momentum operator of an interacting point-form quantum field theory as in Eq. (3) [15]. The vertex operators $\hat{K}$ and $\hat{K}^\dagger$ in Eq. (2), which are responsible for photon emission and absorption, can therefore not directly be taken from point-form quantum electrodynamics. One rather has to make the approximation that the total 4-velocity of the system is conserved at the electromagnetic vertices to end up with a Bakamjian-Thomas type mass operator. In Ref. [12] it has been demonstrated in some detail that this is a way to implement general field theoretical vertex interactions into a Bakamjian-Thomas type framework. If we denote the velocity states of the eM and $eM\gamma$ systems by $|v;\vec{k}_e,\mu_e;\vec{k}_M\rangle$ and $|v;\vec{k}_e,\mu_e;\vec{k}_M;\vec{k}_\gamma,\mu_\gamma\rangle$, respectively, the (velocity conserving) electromagnetic vertex interaction takes on the form [12]

$$\langle v';\vec{k}_e',\mu_e';\vec{k}_M';\vec{k}_\gamma',\mu_\gamma'|\,\hat{K}\,|v;\vec{k}_e,\mu_e;\vec{k}_M\rangle = \langle v;\vec{k}_e,\mu_e;\vec{k}_M|\,\hat{K}^\dagger\,|v';\vec{k}_e',\mu_e';\vec{k}_M';\vec{k}_\gamma',\mu_\gamma'\rangle^*$$
$$= v_0\,\delta^3(\vec{v}'-\vec{v})\,\frac{(2\pi)^3}{\sqrt{(\omega_{k_e'}+\omega_{k_M'}+\omega_{k_\gamma'})^3}\sqrt{(\omega_{k_e}+\omega_{k_M})^3}}$$
$$\times\langle v';\vec{k}_e',\mu_e';\vec{k}_M';\vec{k}_\gamma',\mu_\gamma'|\left(f(\Delta m)\,\hat{\mathcal{L}}^{M\gamma}_{\mathrm{int}}(0)+\hat{\mathcal{L}}^{e\gamma}_{\mathrm{int}}(0)\right)|v;\vec{k}_e,\mu_e;\vec{k}_M\rangle\,, \tag{9}$$

with $\hat{\mathcal{L}}^{M\gamma}_{\mathrm{int}}(x)$ and $\hat{\mathcal{L}}^{e\gamma}_{\mathrm{int}}(x)$ representing the usual interaction densities for scalar and spinor quantum electrodynamics [16]. This is a very natural way to introduce field theoretical vertex interactions into the Bakamjian-Thomas framework, but as a drawback the mass operator does not cluster properly. But as will be shown, if the invariant mass of the electron-meson

system is made sufficiently large, the effect of the wrong cluster separability property is eliminated. The vertex form factor $f(\Delta m)$ depends on the magnitude of the difference of the invariant masses in the initial and final states, $\Delta m = |\,\omega_{k_e} + \omega_{k_M} - \omega_{k'_e} - \omega_{k'_M} - \omega_{k'_\gamma}|$. It is introduced at this place to account for the composite nature of the meson. Moreover, it also serves to partly compensate for the neglect of the off-diagonal terms in the 4-velocity and to regulate the integrals if necessary. But since this is not our primary goal, we do not introduce a second form factor at the electron-photon vertex. After evaluation of the matrix elements on the right-hand side of Eq. (9) the matrix elements of the vertex operator become [17]

$$
\begin{aligned}
&\langle v'; \vec{k}'_e, \mu'_e; \vec{k}'_M; \vec{k}'_\gamma, \mu'_\gamma |\, \hat{K} | v; \vec{k}_e, \mu_e; \vec{k}_M \rangle \\
&= \; v_0\, \delta^3(\vec{v}' - \vec{v}) \frac{(2\pi)^3}{\sqrt{(\omega_{k'_e} + \omega_{k'_M} + \omega_{k'_\gamma})^3}\sqrt{(\omega_{k_e} + \omega_{k_M})^3}} (-1) \\
&\quad \times \Big[Q_e\, \bar{u}_{\mu'_e}(\vec{k}'_e) \gamma_\nu u_{\mu_e}(\vec{k}_e)\, \epsilon^\nu(\vec{k}'_\gamma, \mu'_\gamma)\, (2\pi)^3 2\omega_{k_M} \delta^3(\vec{k}'_M - \vec{k}_M) \\
&\qquad + Q_M\, f(\Delta m)\, (k'_M + k_M)_\nu\, \epsilon^\nu(\vec{k}'_\gamma, \mu'_\gamma)\, (2\pi)^3 2\omega_{k_e} \delta^3(\vec{k}'_e - \vec{k}_e) \Big] \,,
\end{aligned}
\tag{10}
$$

where $\epsilon(\vec{k}_\gamma, \mu_\gamma)$ represent appropriately orthonormalized photon polarization vectors which satisfy the completeness relation

$$
\sum_{\mu_\gamma=0}^{3} \epsilon_\mu(\vec{k}_\gamma, \mu_\gamma)(-g^{\mu_\gamma\mu_\gamma})\, \epsilon^*_\nu(\vec{k}_\gamma, \mu_\gamma) = -g_{\mu\nu} \,. \tag{11}
$$

Note that due to this definition of photon polarization states the orthogonality and completeness relations, Eqs. (5) and (6), have to be modified for each occurring photon such that $\delta_{\mu_\gamma \mu'_\gamma} \to (-g^{\mu_\gamma \mu'_\gamma})$ and $\sum_{\mu_\gamma} \to \sum_{\mu_\gamma}(-g^{\mu_\gamma\mu_\gamma})$.

What we want to calculate are matrix elements of the optical potential $\hat{V}_{\text{opt}}$ between velocity states of the eM system. To this aim we insert the velocity-state completeness relation for the $eM\gamma$ system (cf. Eq. (6)) into Eq. (2) and take matrix elements between

eM velocity states

$$\begin{aligned}
&\langle v'; \vec{k}'_e, \mu'_e; \vec{k}'_M | \, \hat{V}_{\text{opt}}(m) \, | v; \vec{k}_e, \mu_e; \vec{k}_M \rangle \\
&= \langle v'; \vec{k}'_e, \mu'_e; \vec{k}'_M | \, \hat{K}^\dagger \left(\hat{M}_{eM\gamma} - m \right)^{-1} \mathbb{1}''_{eM\gamma} \, \hat{K} | v; \vec{k}_e, \mu_e; \vec{k}_M \rangle \qquad (12) \\
&= \sum_{\mu''_e, \mu''_\gamma} \int \frac{d^3 v''}{(2\pi)^3 v''_0} \int \frac{d^3 k''_e}{(2\pi)^3 2\omega_{k''_e}} \int \frac{d^3 k''_M}{(2\pi)^3 2\omega_{k''_M}} \frac{(\omega_{k''_e} + \omega_{k''_M} + \omega_{k''_\gamma})^3}{2\omega_{k''_\gamma}} \\
&\quad \times \langle v'; \vec{k}'_e, \mu'_e; \vec{k}'_M | \, \hat{K}^\dagger \, | v''; \vec{k}''_e, \mu''_e; \vec{k}''_M; \vec{k}''_\gamma, \mu''_\gamma \rangle \\
&\quad \times \left(\omega_{k''_e} + \omega_{k''_M} + \omega_{k''_\gamma} - m \right)^{-1} \langle v''; \vec{k}''_e, \mu''_e; \vec{k}''_M; \vec{k}''_\gamma, \mu''_\gamma | \, \hat{K} | v; \vec{k}_e, \mu_e; \vec{k}_M \rangle \, .
\end{aligned}$$

Here we have exploited that $|v''; \vec{k}''_e, \mu''_e; \vec{k}''_M; \vec{k}''_\gamma, \mu''_\gamma\rangle$ are eigenstates of the free $eM\gamma$ mass operator

$$\hat{M}_{eM\gamma} | \, v''; \vec{k}''_e, \mu''_e; \vec{k}''_M; \vec{k}''_\gamma, \mu''_\gamma \rangle = (\omega_{k''_e} + \omega_{k''_M} + \omega_{k''_\gamma}) | \, v''; \vec{k}''_e, \mu''_e; \vec{k}''_M; \vec{k}''_\gamma, \mu''_\gamma \rangle \, . \qquad (13)$$

If one now makes use of Eq. (10), a corresponding relation for $\hat{K}^\dagger$, and the completeness relation, Eq. (11), for the photon polarization vectors, the integrals in Eq. (12) can be done by means of the delta functions and one ends up with

$$\begin{aligned}
&\langle v'; \vec{k}'_e, \mu'_e; \vec{k}'_M | \, \hat{V}_{\text{opt}}(m) \, | v; \vec{k}_e, \mu_e; \vec{k}_M \rangle \\
&= v_0 \delta^3(\vec{v}\,' - \vec{v}\,) \frac{(2\pi)^3}{\sqrt{(\omega_{k'_e} + \omega_{k'_M})^3} \sqrt{(\omega_{k_e} + \omega_{k_M})^3}} \\
&\quad \times Q_e \, \bar{u}_{\mu'_e}(\vec{k}'_e) \gamma_\mu u_{\mu_e}(\vec{k}_e) \, \frac{(-g^{\mu\nu})}{2\omega_{k''_\gamma}} \, Q_M (k'_M + k_M)_\nu \\
&\quad \times \left[\frac{f(\Delta m)}{\omega_{k'_M} + \omega_{k_e} + \omega_{k''_\gamma} - m} + \frac{f(\Delta m')}{\omega_{k_M} + \omega_{k'_e} + \omega_{k''_\gamma} - m} \right] \, . \qquad (14)
\end{aligned}$$

Here $\Delta m = | \, \omega_{k_M} + \omega_{k_e} - \omega_{k'_M} - \omega_{k_e} - \omega_{k''_\gamma} |$ and $\Delta m' = | \, \omega_{k_M} + \omega_{k'_e} + \omega_{k''_\gamma} - \omega_{k'_M} - \omega_{k'_e} |$, respectively. The two terms between square brackets correspond to the two possible time orderings, i.e. photon emission by the meson with subsequent absorption by the electron and vice versa. In evaluating the matrix elements of $\hat{V}_{\text{opt}}$ we have neglected self-energy contributions in which the photon is emitted and absorbed by the same particle. Although all double-primed variables should be eliminated by now, we have kept $\omega_{k''_\gamma}$ for better readability. Since $\vec{k}''_\gamma = \pm(\vec{k}'_e - \vec{k}_e)$ differs only in the sign for the two time orderings, $\omega_{k''_\gamma} = | \, (\vec{k}'_e - \vec{k}_e) |$ is independent of the time ordering.

Eq. (14) represents a general matrix element of the one-photon-exchange optical potential. Electromagnetic hadron form factors, however, are usually extracted from the elastic

electron-hadron scattering amplitude, calculated in the one-photon-exchange approximation. This means that we can restrict our considerations to on-shell matrix elements of $\hat{V}_{\rm opt}$ for which $m = \omega_{k_M} + \omega_{k_e} = \omega_{k'_M} + \omega_{k'_e}$ and $\omega_{k_M} = \omega_{k'_M}$, $\omega_{k_e} = \omega_{k'_e}$ [36]. As a consequence we have $\Delta m = \Delta m' = \omega_{k''_\gamma} = |\vec{k}'_M - \vec{k}_M| = \sqrt{-q_\mu q^\mu} = Q$, with $q^\mu = k'_M - k_M$ being the 4-momentum transfer between incoming and outgoing meson. With these relations the two terms in square brackets can be combined and the on-shell matrix elements of the optical potential can be expressed as a contraction of the electromagnetic electron current

$$j_\mu(\vec{k}'_e, \mu'_e; \vec{k}_e, \mu_e) = Q_e\, \bar{u}_{\mu'_e}(\vec{k}'_e)\gamma_\mu u_{\mu_e}(\vec{k}_e) \tag{15}$$

with an electromagnetic meson current

$$J_\nu^{\rm phen}(\vec{k}'_M; \vec{k}_M) = Q_M\, (k'_M + k_M)_\nu\, f(Q) = J_\nu^{\rm point}(\vec{k}'_M; \vec{k}_M)\, f(Q) \tag{16}$$

multiplied with the covariant photon propagator $-g^{\mu\nu}/Q^2$ (and a kinematical factor),

$$\begin{aligned} &\langle v'; \vec{k}'_e, \mu'_e; \vec{k}'_M |\, \hat{V}_{\rm opt}(m)\, | v; \vec{k}_e, \mu_e; \vec{k}_M\rangle_{\rm on-shell} \\ &= v_0 \delta^3(\vec{v}' - \vec{v}\,)\, \frac{(2\pi)^3}{\sqrt{(\omega_{k'_e} + \omega_{k'_M})^3}\sqrt{(\omega_{k_e} + \omega_{k_M})^3}}\, j_\mu(\vec{k}'_e, \mu'_e; \vec{k}_e, \mu_e)\, \frac{(-g^{\mu\nu})}{Q^2}\, J_\nu^{\rm phen}(\vec{k}'_M; \vec{k}_M)\,. \end{aligned} \tag{17}$$

Apart from the kinematical factor in front, the right-hand side of Eq. (17) corresponds to the familiar one-photon exchange amplitude for elastic electron-meson scattering (calculated in the electron-meson center-of-mass system). It is therefore justified to interpret $f(\Delta m)$ in Eq. (16) as the electromagnetic meson form factor which parameterizes the composite nature of the meson in a phenomenological way.

At this point a warning is in order. At first sight one may get the impression that the currents defined in Eqs. (15) and (16) transform like 4-vectors. This is not the case, since the momenta which appear in the currents are particle momenta in the center of mass of the electron-meson system. The effect of a Lorentz transformation on such momenta is just a Wigner rotation, as can be seen from Eq. (7). Currents with the correct covariance properties, however, can be obtained by going back to the physical particle momenta by means of a canonical boost with velocity $\vec{v}$, i.e. $p_i^{(\prime)} = B_c(v) k_i^{(\prime)}$ with $i = e, M$. If we define the electron and meson currents for the physical particle momenta by

$$j_\mu(\vec{p}'_e, \sigma'_e; \vec{p}_e, \sigma_e) := (B_c(v))_\mu{}^\rho\, j_\rho(\vec{k}'_e, \mu'_e; \vec{k}_e, \mu_e)\, D^{1/2\,*}_{\mu'_e\sigma'_e}(R_{\rm W}^{-1}(\frac{k'_e}{m_e}, B_c(v)))\, D^{1/2}_{\mu_e\sigma_e}(R_{\rm W}^{-1}(\frac{k_e}{m_e}, B_c(v))) \tag{18}$$

and

$$J_\nu^{\text{phen}}(\vec{p}_M^{\,\prime};\vec{p}_M) := (B_c(v))_\nu{}^\rho \, J_\rho^{\text{phen}}(\vec{k}_M^{\,\prime};\vec{k}_M) \tag{19}$$

it can be checked that these currents exhibit the correct covariance properties under Lorentz transformations and are obviously conserved.

B. Constituent level

Our next objective will be to determine $f(\Delta m)$ starting from a microscopic constituent-quark model for the meson. We will proceed in a similar way as before with the only difference, that the mass operator of our coupled channel system is now defined on a Hilbert space which is the direct sum of channel Hilbert spaces consisting of electron-quark-antiquark $|\psi_{eq\bar{q}}\rangle$ and electron-quark-antiquark-photon states $|\psi_{eq\bar{q}\gamma}\rangle$. Furthermore, the quark-antiquark pair is assumed to be confined, so that the channel mass operators $\hat{M}_{eM}$ and $\hat{M}_{eM\gamma}$ in Eqs. (1) and (2) become

$$\hat{M}_{eM} \rightarrow \hat{M}_{eC} = \hat{M}_{eq\bar{q}} + \hat{V}_{\text{conf}}\,, \qquad \hat{M}_{eM\gamma} \rightarrow \hat{M}_{eC\gamma} = \hat{M}_{eq\bar{q}\gamma} + \hat{V}_{\text{conf}}\,, \tag{20}$$

where $\hat{M}_{eq\bar{q}}$ and $\hat{M}_{eq\bar{q}\gamma}$ are free mass operators for the electron-quark-antiquark and electron-quark-antiquark-photon systems and $\hat{V}_{\text{conf}}$ is an instantaneous confinement potential acting between quark and antiquark. The subscript "C" of $\hat{M}_{eC}$ and $\hat{M}_{eC\gamma}$ should indicate that the channel mass operators provide already a clustering of the quark-antiquark subsystem. On the constituent level the vertex operators $\hat{K}$ and $\hat{K}^\dagger$ should describe the emission and absorption of a photon by a pointlike (anti)quark. To distinguish them from the hadronic case we will denote them by $\hat{K}_{q\gamma}$ and $\hat{K}_{q\gamma}^\dagger$.

On the constituent level it is advantageous to introduce two sets of basis states. One set consists of velocity states that are eigenstates of the free channel mass operators,

$$\begin{aligned}
&\hat{M}_{eq\bar{q}}\,|\,v;\vec{k}_e,\mu_e;\vec{k}_q,\mu_q;\vec{k}_{\bar{q}},\mu_{\bar{q}}\rangle = (\omega_{k_e}+\omega_{k_q}+\omega_{k_{\bar{q}}})|\,v;\vec{k}_e,\mu_e;\vec{k}_q,\mu_q;\vec{k}_{\bar{q}},\mu_{\bar{q}}\rangle\,,\\
&\hat{M}_{eq\bar{q}\gamma}\,|\,v;\vec{k}_e,\mu_e;\vec{k}_q,\mu_q;\vec{k}_{\bar{q}},\mu_{\bar{q}};\vec{k}_\gamma,\mu_\gamma\rangle\\
&\qquad = (\omega_{k_e}+\omega_{k_q}+\omega_{k_{\bar{q}}}+\omega_{k_\gamma})|\,v;\vec{k}_e,\mu_e;\vec{k}_q,\mu_q;\vec{k}_{\bar{q}},\mu_{\bar{q}};\vec{k}_\gamma,\mu_\gamma\rangle\,,
\end{aligned} \tag{21}$$

the other set consists of velocity states that are eigenstates of the channel mass operators

with confinement potential

$$
\begin{aligned}
\hat{M}_{eC}\,|\,\underline{v};\vec{\underline{k}}_e,\underline{\mu}_e;\vec{\underline{k}}_C,n,j,\tilde{m}_j,[\tilde{l},\tilde{s}]\rangle &= (\omega_{\underline{k}_e}+\omega_{\underline{k}_C})|\,\underline{v};\vec{\underline{k}}_e,\underline{\mu}_e;\vec{\underline{k}}_C,n,j,\tilde{m}_j,[\tilde{l},\tilde{s}]\rangle\,, \\
\hat{M}_{eC\gamma}\,|\,\underline{v};\vec{\underline{k}}_e,\underline{\mu}_e;\vec{\underline{k}}_C,n,j,\tilde{m}_j,[\tilde{l},\tilde{s}];\vec{\underline{k}}_\gamma,\underline{\mu}_\gamma\rangle & \\
= (\omega_{\underline{k}_e}+\omega_{\underline{k}_C}+\omega_{\underline{k}_\gamma})|\,\underline{v};\vec{\underline{k}}_e,\underline{\mu}_e;\vec{\underline{k}}_C,n,j,\tilde{m}_j,[\tilde{l},\tilde{s}];\vec{\underline{k}}_\gamma,\underline{\mu}_\gamma\rangle\,. &
\end{aligned}
\tag{22}
$$

To distinguish these two sets of basis states we have underlined velocities, momenta and spin projections for the velocity states of $q\bar{q}$ clusters. $(n,j,\tilde{m}_j,[\tilde{l},\tilde{s}])$ are the discrete quantum numbers of the $q\bar{q}$ cluster labeling orbital excitation, total angular momentum, its projection, orbital angular momentum, and total spin, respectively. The tilde should indicate that the corresponding quantum numbers are defined in the rest frame of the $q\bar{q}$ subsystem. Due to the occurrence of the discrete quantum numbers $(n,j,\tilde{m}_j,[\tilde{l},\tilde{s}])$ in cluster velocity states the orthogonality and completeness relations, Eqs. (5) and (6), have to be supplemented by corresponding sums and Kronecker deltas [18].

The quantities that can be directly compared with Eq. (14) are on-shell matrix elements of the (constituent-level) optical potential $\hat{V}^{\mathrm{const}}_{\mathrm{opt}}$ between cluster velocity states with the cluster possessing the quantum numbers of the meson. For a pseudoscalar meson one thus has to calculate

$$
\begin{aligned}
&\langle\underline{v}';\vec{\underline{k}}'_e,\underline{\mu}'_e;\vec{\underline{k}}'_C,n,0,0,[0,0]|\,\hat{V}^{\mathrm{const}}_{\mathrm{opt}}(m)|\,\underline{v};\vec{\underline{k}}_e,\underline{\mu}_e;\vec{\underline{k}}_C,n,0,0,[0,0]\rangle_{\mathrm{on-shell}} \\
&= \langle\underline{v}';\vec{\underline{k}}'_e,\underline{\mu}'_e;\vec{\underline{k}}'_C,n,0,0,[0,0]|\,\hat{K}^\dagger_{q\gamma}\left(\hat{M}_{eC\gamma}-m\right)^{-1}\hat{K}_{q\gamma}|\,\underline{v};\vec{\underline{k}}_e,\underline{\mu}_e;\vec{\underline{k}}_C,n,0,0,[0,0]\rangle\,,
\end{aligned}
\tag{23}
$$

with $m=\omega_{\underline{k}_e}+\omega_{\underline{k}_C}=\omega_{\underline{k}'_e}+\omega_{\underline{k}'_C}$ and $\omega_{\underline{k}_e}=\omega_{\underline{k}'_e}$ and $\omega_{\underline{k}_C}=\omega_{\underline{k}'_C}$. This can again be done by inserting completeness relations for free and cluster velocity states at appropriate places

$$
\begin{aligned}
&\langle\underline{v}';\vec{\underline{k}}'_e,\underline{\mu}'_e;\vec{\underline{k}}'_C,n,0,0,[0,0]|\,\hat{V}^{\mathrm{const}}_{\mathrm{opt}}(m)|\,\underline{v};\vec{\underline{k}}_e,\underline{\mu}_e;\vec{\underline{k}}_C,n,0,0,[0,0]\rangle_{\mathrm{on-shell}} \\
&= \langle\underline{v}';\vec{\underline{k}}'_e,\underline{\mu}'_e;\vec{\underline{k}}'_C,n,0,0,[0,0]|\,\mathbb{1}'_{eq\bar{q}}\hat{K}^\dagger_{q\gamma}\mathbb{1}'''_{eq\bar{q}\gamma}\left(\hat{M}_{eC\gamma}-m\right)^{-1}\mathbb{1}''_{eC\gamma} \\
&\quad\times\mathbb{1}''_{eq\bar{q}\gamma}\hat{K}_{q\gamma}\mathbb{1}_{eq\bar{q}}\,|\,\underline{v};\vec{\underline{k}}_e,\underline{\mu}_e;\vec{\underline{k}}_C,n,0,0,[0,0]\rangle\,.
\end{aligned}
\tag{24}
$$

The matrix elements one needs to know are therefore typically scalar products between free and cluster velocity states as well as matrix elements of the vertex operators between free velocity states. With our normalization of velocity states, cf. Eq. (5), such matrix elements

are given by (see also [17, 18])

$$
\begin{aligned}
&\langle v;\vec{k}_e,\mu_e;\vec{k}_q,\mu_q;\vec{k}_{\bar q},\mu_{\bar q}|\,\underline{v};\underline{\vec{k}}_e,\underline{\mu}_e;\underline{\vec{k}}_C,n,j,\tilde m_j,[\tilde l,\tilde s]\rangle\\
&= (2\pi)^{15/2}\,\underline{v}_0\,\delta^3(\vec v-\underline{\vec v})\,\delta^3(\vec k_e-\underline{\vec k}_e)\,\delta_{\mu_e\underline{\mu}_e}\sqrt{\frac{2\omega_{\underline{k}_e}2\omega_{\underline{k}_C}}{(\omega_{\underline{k}_e}+\omega_{\underline{k}_C})^3}}\sqrt{\frac{2\omega_{k_e}2\omega_{k_{q\bar q}}}{(\omega_{k_e}+\omega_{k_{q\bar q}})^3}}\sqrt{\frac{2\omega_{\tilde k_q}2\omega_{\tilde k_{\bar q}}}{(\omega_{\tilde k_q}+\omega_{\tilde k_{\bar q}})^3}}\\
&\quad\times\sum_{\tilde m_l=-\tilde l}^{\tilde l}\sum_{\tilde m_s=-\tilde s}^{\tilde s}\sum_{\tilde\mu_q,\tilde\mu_{\bar q}=\pm1/2}C^{j\tilde m_j}_{\tilde l\tilde m_l\tilde s\tilde m_s}\,C^{\tilde s\tilde m_s}_{\frac12\tilde\mu_q\frac12\tilde\mu_{\bar q}}\,D^{1/2}_{\mu_q\tilde\mu_q}(R_{\mathrm W}(\frac{\tilde k_q}{m_q},B_c(v_{q\bar q})))\,D^{1/2}_{\mu_{\bar q}\tilde\mu_{\bar q}}(R_{\mathrm W}(\frac{\tilde k_{\bar q}}{m_{\bar q}},B_c(v_{q\bar q})))\\
&\quad\times u_{n\tilde l}\,(|\vec{\tilde k}_q|)\,Y_{\tilde l\tilde m_l}(\hat{\tilde k}_q)\,,
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
&\langle v;\vec{k}_e,\mu_e;\vec{k}_q,\mu_q;\vec{k}_{\bar q},\mu_{\bar q};\vec k_\gamma,\mu_\gamma|\,\underline{v};\underline{\vec{k}}_e,\underline{\mu}_e;\underline{\vec{k}}_C,n,j,\tilde m_j,[\tilde l,\tilde s];\underline{\vec k}_\gamma,\underline{\mu}_\gamma\rangle\\
&= (2\pi)^{21/2}\,\underline{v}_0\,\delta^3(\vec v-\underline{\vec v})\,\delta^3(\vec k_e-\underline{\vec k}_e)\,\delta_{\mu_e\underline{\mu}_e}\,\delta^3(\vec k_\gamma-\underline{\vec k}_\gamma)\,(-g_{\mu_\gamma\underline{\mu}_\gamma})\\
&\quad\times\sqrt{\frac{2\omega_{\underline{k}_e}2\omega_{\underline{k}_C}2\omega_{\underline{k}_\gamma}}{(\omega_{\underline{k}_e}+\omega_{\underline{k}_C}+\omega_{\underline{k}_\gamma})^3}}\sqrt{\frac{2\omega_{k_e}2\omega_{k_{q\bar q}}2\omega_{k_\gamma}}{(\omega_{k_e}+\omega_{k_{q\bar q}}+\omega_{k_\gamma})^3}}\sqrt{\frac{2\omega_{\tilde k_q}2\omega_{\tilde k_{\bar q}}}{(\omega_{\tilde k_q}+\omega_{\tilde k_{\bar q}})^3}}\\
&\quad\times\sum_{\tilde m_l=-\tilde l}^{\tilde l}\sum_{\tilde m_s=-\tilde s}^{\tilde s}\sum_{\tilde\mu_q,\tilde\mu_{\bar q}=\pm1/2}C^{j\tilde m_j}_{\tilde l\tilde m_l\tilde s\tilde m_s}\,C^{\tilde s\tilde m_s}_{\frac12\tilde\mu_q\frac12\tilde\mu_{\bar q}}\,D^{1/2}_{\mu_q\tilde\mu_q}(R_{\mathrm W}(\frac{\tilde k_q}{m_q},B(v_{q\bar q})))\,D^{1/2}_{\mu_{\bar q}\tilde\mu_{\bar q}}(R_{\mathrm W}(\frac{\tilde k_{\bar q}}{m_{\bar q}},B(v_{q\bar q})))\\
&\quad\times u_{n\tilde l}\,(|\vec{\tilde k}_q|)\,Y_{\tilde l\tilde m_l}(\hat{\tilde k}_q)\,,
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
&\langle v';\vec k'_e,\mu'_e;\vec k'_q,\mu'_q;\vec k'_{\bar q},\mu'_{\bar q};\vec k'_\gamma,\mu'_\gamma|\,\hat K\,|v;\vec k_e,\mu_e;\vec k_q,\mu_q;\vec k_{\bar q},\mu_{\bar q}\rangle\\
&= v_0\,\delta^3(\vec v'-\vec v)\,\frac{(2\pi)^3}{\sqrt{(\omega_{k'_e}+\omega_{k'_q}+\omega_{k'_{\bar q}}+\omega_{k'_\gamma})^3}\sqrt{(\omega_{k_e}+\omega_{k_q}+\omega_{k_{\bar q}})^3}}\\
&\quad\times\langle v';\vec k'_e,\mu'_e;\vec k'_q,\mu'_q;\vec k'_{\bar q},\mu'_{\bar q};\vec k'_\gamma,\mu'_\gamma|\,\left(\hat{\mathcal L}^{e\gamma}_{\mathrm{int}}(0)+\hat{\mathcal L}^{q\gamma}_{\mathrm{int}}(0)\right)|v;\vec k_e,\mu_e;\vec k_q,\mu_q;\vec k_{\bar q},\mu_{\bar q}\rangle\\
&= v_0\,\delta^3(\vec v'-\vec v)\,\frac{(2\pi)^3}{\sqrt{(\omega_{k'_e}+\omega_{k'_M}+\omega_{k'_\gamma})^3}\sqrt{(\omega_{k_e}+\omega_{k_M})^3}}(-1)\\
&\quad\times\Big[\,Q_e\,\bar u_{\mu'_e}(\vec k'_e)\gamma_\nu u_{\mu_e}(\vec k_e)\,\epsilon^\nu(\vec k'_\gamma,\mu'_\gamma)\,(2\pi)^3 2\omega_{k_q}\delta^3(\vec k'_q-\vec k_q)\,(2\pi)^3 2\omega_{k_{\bar q}}\delta^3(\vec k'_{\bar q}-\vec k_{\bar q})\\
&\qquad+Q_q\,\bar u_{\mu'_q}(\vec k'_q)\gamma_\nu u_{\mu_q}(\vec k_q)\,\epsilon^\nu(\vec k'_\gamma,\mu'_\gamma)\,(2\pi)^3 2\omega_{k_e}\delta^3(\vec k'_e-\vec k_e)\,(2\pi)^3 2\omega_{k_{\bar q}}\delta^3(\vec k'_{\bar q}-\vec k_{\bar q})\\
&\qquad+Q_{\bar q}\,\bar v_{\mu_{\bar q}}(\vec k_{\bar q})\gamma_\nu v_{\mu'_{\bar q}}(\vec k_{\bar q})\,\epsilon^\nu(\vec k'_\gamma,\mu'_\gamma)\,(2\pi)^3 2\omega_{k_e}\delta^3(\vec k'_e-\vec k_e)\,(2\pi)^3 2\omega_{k_q}\delta^3(\vec k'_q-\vec k_q)\Big]\,,
\end{aligned}
\tag{27}
$$

and the hermitian conjugate expressions, respectively. $\hat{\mathcal L}^{q\gamma}_{\mathrm{int}}(x)$ is again the usual interaction density for spinor quantum electrodynamics describing the coupling of the photon to a quark or antiquark. Spins and angular momenta are coupled in the rest frame of the $q\bar q$ subsystem. This is the reason that the Wigner D functions $D^{1/2}_{\mu\tilde\mu}(R_W)$ relating the spin projection of the

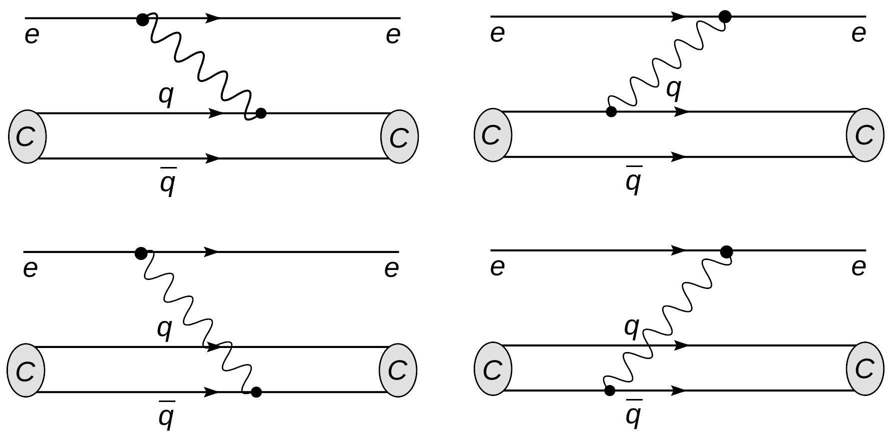

FIG. 1: Graphical representation of the four contributions to the one-photon-exchange optical potential for the scattering of an electron by a quark-antiquark cluster.

(anti)quark in the $q\bar{q}$ rest frame to the spin projection in the overall $eq\bar{q}(\gamma)$ center-of-mass frame appear in Eqs. (25) and (26). The pertinent Wigner rotation is given by Eq. (8) with

$$v_{q\bar{q}} = \frac{k_{q\bar{q}}}{m_{q\bar{q}}} = \frac{k_q + k_{\bar{q}}}{m_{q\bar{q}}} \quad \text{and} \quad m_{q\bar{q}} = \tilde{k}_q^0 + \tilde{k}_{\bar{q}}^0 = \sqrt{(k_q^0 + k_{\bar{q}}^0)^2 - (\vec{k}_q + \vec{k}_{\bar{q}})^2}\,. \tag{28}$$

For later purposes we note that $\vec{k}_e + \vec{k}_q + \vec{k}_{\bar{q}} = \vec{k}_e + \underline{\vec{k}}_C = 0$ and hence $\vec{k}_q + \vec{k}_{\bar{q}} = \underline{\vec{k}}_C$ so that $\vec{v}_{q\bar{q}} = \underline{\vec{k}}_C / m_{q\bar{q}}$. The $u_{n\tilde{l}}(|\vec{\tilde{k}}_q|)$ form a complete set of radial wave functions for the confined quark-antiquark pair orthonormalized according to

$$\int_0^\infty d\tilde{k}\, \tilde{k}^2\, u_{n'\tilde{l}'}(\tilde{k}) u_{n\tilde{l}}(\tilde{k}) = \delta_{n'n}\, \delta_{\tilde{l}'\tilde{l}}\,. \tag{29}$$

The $Y_{\tilde{l}\tilde{m}_l}(\hat{\tilde{k}})$ are usual spherical harmonics depending on $\hat{\tilde{k}} = \vec{\tilde{k}}/|\vec{\tilde{k}}|$. In Eqs. (25) and (26) it has been assumed for simplicity that the confining potential is spin independent.

With these results for the matrix elements most of the sums and integrals on the right-hand side of Eq. (24) (coming from the inserted completeness relations) can be carried out analytically by means of Dirac and Kronecker deltas. If self-energy contributions are again neglected the optical potential consists of four terms which are depicted in Fig. 1. Here the blobs which connect the quark and antiquark lines symbolize integrals over wave functions of the incoming and outgoing quark-antiquark cluster. These integrals are the same for both time orderings, i.e. for the absorption and emission of the photon by the (anti)quark. Thus both time orderings can be combined, as on the hadronic level, and one is left with a photon-exchange contribution for the quark and another for the antiquark. If, in addition,

quark and antiquark have the same mass $m_q = m_{\bar{q}}$, even the integrals for the quark and antiquark contributions become the same so that the final expression for the one-photon-exchange optical potential takes on a simple form on the constituent level comparable to that on the hadronic level (cf. Eq. (17)):

$$\begin{aligned}
&\langle \underline{v}'; \underline{\vec{k}}'_e, \underline{\mu}'_e; \underline{\vec{k}}'_C, n, 0, 0, [0,0] | \hat{V}^{\rm const}_{\rm opt}(m) | \underline{v}; \underline{\vec{k}}_e, \underline{\mu}_e; \underline{\vec{k}}_C, n, 0, 0, [0,0] \rangle_{\rm on-shell} \\
&= \underline{v}_0 \delta^3(\underline{\vec{v}}' - \underline{\vec{v}}) \frac{(2\pi)^3}{\sqrt{(\omega_{\underline{k}'_e} + \omega_{\underline{k}'_C})^3} \sqrt{(\omega_{\underline{k}_e} + \omega_{\underline{k}_C})^3}} j_\mu(\underline{\vec{k}}'_e, \underline{\mu}'_e; \underline{\vec{k}}_e, \underline{\mu}_e) \frac{(-g^{\mu\nu})}{Q^2} J^{\rm micro}_\nu(\underline{\vec{k}}'_C; \underline{\vec{k}}_C)\,,
\end{aligned} \tag{30}$$

with the (preliminary) microscopic meson current being given by

$$\begin{aligned}
J^{\rm micro}_\nu(\underline{\vec{k}}'_C; \underline{\vec{k}}_C) &= (Q_q + Q_{\bar{q}}) \sqrt{\omega_{\underline{k}_C} \omega_{\underline{k}'_C}} \int \frac{d^3 \tilde{k}'_q}{\omega_{k_q}} \sqrt{\frac{\omega_{\tilde{k}_q}}{\omega_{\tilde{k}'_q}}} \sqrt{\frac{\omega_{k_{q\bar{q}}}}{\omega_{k'_{q\bar{q}}}}} \\
&\times u^*_{n0}(|\vec{\tilde{k}}'_q|)\, Y^*_{00}(\hat{\tilde{k}}'_q)\, u_{n0}(|\vec{\tilde{k}}_q|)\, Y_{00}(\hat{\tilde{k}}_q) \Bigg[\sum_{\mu_q, \mu'_q = \pm\frac{1}{2}} \bar{u}_{\mu'_q}(\vec{k}'_q) \gamma_\nu u_{\mu_q}(\vec{k}_q) \\
&\times D^{1/2}_{\mu_q \mu'_q} \left(R_{\rm W}(\frac{\tilde{k}_q}{m_q}, B_c(v_{q\bar{q}}))\, R^{-1}_{\rm W}(\frac{\tilde{k}_{\bar{q}}}{m_{\bar{q}}}, B_c(v_{q\bar{q}}))\, R_{\rm W}(\frac{\tilde{k}'_q}{m_q}, B_c(v'_{q\bar{q}}))\, R^{-1}_{\rm W}(\frac{\tilde{k}'_{\bar{q}}}{m_{\bar{q}}}, B_c(v'_{q\bar{q}})) \right) \Bigg]\,.
\end{aligned} \tag{31}$$

According to our notation introduced previously, momenta without and with tilde satisfy $\vec{k}_e + \vec{k}_q + \vec{k}_{\bar{q}} = 0$ and $\vec{\tilde{k}}_q + \vec{\tilde{k}}_{\bar{q}} = 0$, respectively. These two sets of momenta are connected via the canonical spin boost $B_c(v_{q\bar{q}})$, e.g. $B_c(v_{q\bar{q}})\tilde{k}_q = k_q$. An analogous property holds for the primed momenta. Momenta with and without prime are are related by $\vec{k}'_q = \vec{k}_q + (\underline{\vec{k}}'_C - \underline{\vec{k}}_C) = \vec{k}_q + \underline{\vec{k}}_\gamma = \vec{k}_q + \vec{k}_\gamma$. This means that we have 3-momentum conservation at the photon-quark vertex, a property which one would not expect in point-form quantum mechanics. Here it results from the velocity-state representation, in particular the associated center-of-mass kinematics, and the spectator conditions for electron and antiquark. For the physical momenta, i.e. the center-of-mass momenta boosted by $B_c(v)$, none of the 4-momentum components is conserved at the electromagnetic vertices, in general (if $v \neq 0$). As we have seen already on the hadronic level, the denominator of the (covariant) photon propagator (which includes both time orderings) is given by $Q^2 = -\underline{q}_\mu \underline{q}^\mu$ with $\underline{q}^\mu = (\underline{k}'_C - \underline{k}_C)^\mu$ being the 4-momentum transfer between incoming and outgoing cluster [37]. The Wigner D function occurring in Eq. (31) results from combining individual Wigner D functions for the quark and antiquark. Thereby the sums over the spin projections can be carried out with the help of the spectator conditions and the Clebsch-Gordan coefficients $C^{00}_{\frac{1}{2}\tilde{\mu}_q \frac{1}{2}\tilde{\mu}_{\bar{q}}}$ which couple the quark and antiquark spins to zero meson spin.

The warning that we have sounded at the end of the previous subsection has to be repeated here. The microscopic meson current, as defined in Eq. (31) does not behave like a 4-vector under Lorentz transformations Λ. Rather it transforms by the Wigner rotation $R_{\mathrm{W}}(v,\Lambda)$. The current with the correct transformation properties is again the one involving the physical meson momenta, i.e.

$$J_\nu^{\mathrm{micro}}(\vec{p}_C\,';\vec{p}_C) := (B_c(v))_\nu{}^\rho\, J_\rho^{\mathrm{micro}}(\underline{\vec{k}}_C\,';\underline{\vec{k}}_C)\,. \tag{32}$$

In general, the 4-momentum transfer between incoming and outgoing (active) quark $q^\mu = (k_q' - k_q)^\mu$ deviates from the 4-momentum transfer between incoming and outgoing cluster $\underline{q}^\mu = (\underline{k}_C' - \underline{k}_C)^\mu$. Whereas the 3-momentum transfers are the same on the hadronic and on the constituent level, i.e. $\underline{\vec{q}} = \underline{\vec{k}}_C\,' - \underline{\vec{k}}_C = \vec{k}_q' - \vec{k}_q = \vec{q}$, the zero components differ, $\underline{q}^0 \neq q^0$. Due to the center-of-mass kinematics associated with the velocity states $\omega_{\underline{k}_C} = \omega_{\underline{k}_C'}$ and hence $\underline{q}^0 = 0$, but on the other hand $\omega^2_{k_q'} = \omega^2_{k_q} - 2\vec{q}\cdot\vec{k}_{\tilde{q}}'$ so that $q^0 \neq 0$. This means that not all of the 4-momentum transferred to the cluster is also transferred to the active constituent. Nevertheless, the microscopic current $J_\nu^{\mathrm{micro}}(\underline{\vec{k}}_C\,';\underline{\vec{k}}_C)$ and hence also $J_\nu^{\mathrm{micro}}(\vec{p}_C\,';\vec{p}_C)$ is conserved, i.e. $(p_C' - p_C)^\nu J_\nu^{\mathrm{micro}}(\vec{p}_C\,';\vec{p}_C) = 0$. The analytical proof of current conservation amounts to showing that the integral in Eq. (31) vanishes if the spinor product $\bar{u}\gamma_\nu u$ is replaced by $\bar{u}\gamma_0(\omega_q' - \omega_q)u$. By a change of the integration variables $d^3\tilde{k}_q' \to d^3\tilde{k}_q$ it can be shown that the integral over $\bar{u}\gamma_0\omega_q' u$ goes over into the integral over $\bar{u}\gamma_0\omega_q u$ and vice versa so that the difference vanishes.

III. IDENTIFYING THE MESON FORM FACTOR

By comparing the optical one-photon-exchange potentials for electron-meson scattering on the hadronic and the constituent levels we are now in the position to extract the vertex form factor in a unique way. If the cluster has the same (external) quantum numbers as the meson, we can equate the right-hand-sides of Eqs. (17) and (30) to find

$$f(\Delta m, |\vec{k}_M|) = \frac{j^\mu(\vec{k}_e', \mu_e'; \vec{k}_e, \mu_e) J_\mu^{\mathrm{micro}}(\vec{k}_M'; \vec{k}_M)}{j^\mu(\vec{k}_e', \mu_e'; \vec{k}_e, \mu_e) J_\mu^{\mathrm{point}}(\vec{k}_M'; \vec{k}_M)}\,, \tag{33}$$

with J_μ^{point} being defined in Eq. (16). Here we have introduced $|\vec{k}_M|$ as a further argument of the vertex form factor since it is a priori not clear from its microscopic expression, Eq. (33), that it will only depend on Δm. Poincaré invariance of our Bakamjian-Thomas

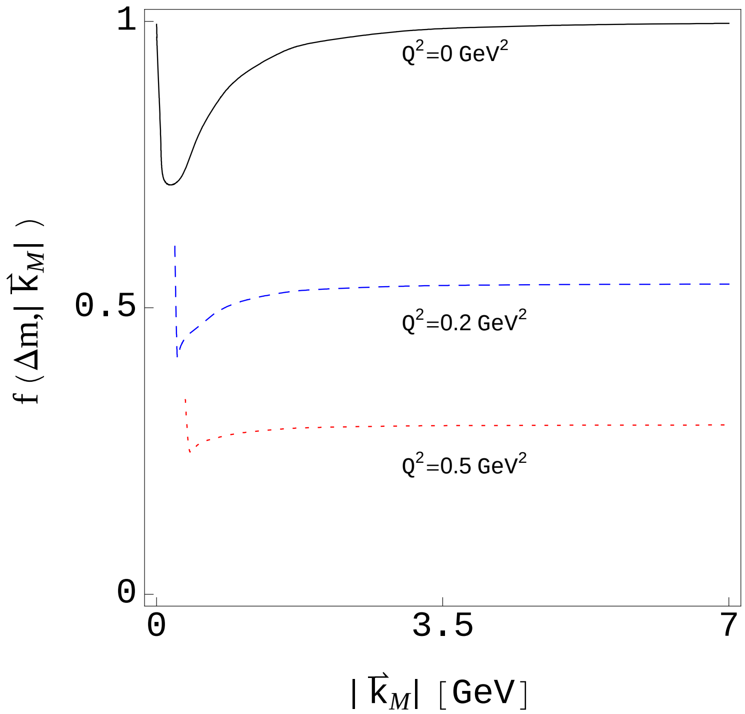

FIG. 2: $|\vec{k}_M|$-dependence of the pion form factor for different values of $Q^2 = \Delta m^2$ ($Q^2 = 0$ GeV^2 solid, $Q^2 = 0.2$ GeV^2 dashed, $Q^2 = 0.5$ GeV^2 dotted) as evaluated by means of Eq. (33).

type approach would not be spoiled by vertex form factors that also depend on the whole set of independent Lorentz invariants involved in the process. For elastic electron-meson scattering these are, e.g., the Mandelstam variables $s = \left(\sqrt{m_e + |\vec{k}_M|^2} + \sqrt{m_M + |\vec{k}_M|^2}\right)^2$ and $t = -\Delta m^2 = -Q^2$.

A reasonable microscopic model for an electromagnetic hadron form factor should, of course, only depend on the momentum transfer $\Delta m = \sqrt{-t}$ and not on $\sqrt{s}$, the invariant mass of the electron-meson system. To check, whether this is the case for our form factor expression, Eq. (33), we make a numerical study with a simple harmonic oscillator wave function for the quark-antiquark bound state:

$$u_{00}(|\vec{\tilde{k}}_q|) = \frac{2}{\pi^{1/4} a^{3/2}} \mathrm{e}^{-\frac{|\vec{\tilde{k}}_q|^2}{2a^2}} \,. \qquad (34)$$

There are just two free parameters in this simple model, the oscillator parameter a and the constituent-quark mass m_q. For later comparison we take these parameters from a paper of Chung, Coester, and Polyzou [19] in which a reasonable fit of the pion form-factor data has been achieved using front-form quantum mechanics by choosing $a = 0.35$ GeV^2 and

$m_q = 0.21$ GeV. We want to emphasize that our goal is not to end up with an optimal fit of the pion form factor, but we rather want to exhibit the virtues of our point-form approach and compare it to other attempts to develop microscopic models for hadron form factors. The dependence of the pion form factor on $|\vec{k}_M|$ (or Mandelstam s) for different values of the momentum transfer Q, as evaluated by means of Eq. (33), is shown in Fig. 2. What one observes is a moderate dependence on $|\vec{k}_M|$ at low energies which vanishes nearly completely for $|\vec{k}_M| > 2$ GeV. In addition, this $|\vec{k}_M|$-dependence becomes weaker with increasing momentum transfer Q^2.

These observations mean that the electromagnetic meson current $J_\nu^{\mathrm{micro}}(\vec{p}_M'; \vec{p}_M)$ (cf. Eq. (32)), which we have derived from the one-photon exchange optical potential, does not have all the properties it should have. It satisfies current conservation and transforms like a 4-vector, but it cannot be written as a sum of covariants times Lorentz invariants (Q^2 and m_M^2) which are solely built from the incoming and outgoing meson 4-momenta (p_M^μ and $p_M^{\mu\prime}$). Actually, this result is not too surprising. Recall that the electromagnetic vertex has been approximated such that electron-meson scattering could be treated within the Bakamjian-Thomas framework. Associated with the Bakamjian-Thomas framework is the problem of cluster separabilty [4]. In our approach this problem is inherent in the definition of the cluster wave functions via velocity states, Eqs. (25) and (26). The wave function of the cluster changes with the presence of additional particles, even if these particles don't interact with the cluster. This change is essentially proportional to the fraction of the cluster binding energy over the invariant mass of the whole electron-meson system.

Therefore the $|\vec{k}_M|$-dependence vanishes rather quickly with increasing invariant mass and it is suggestive to take the limit $|\vec{k}_M| \to \infty$. In this way one ends up with a vertex form factor that depends only on the momentum transfer Q^2. It is even more interesting to see explicitly that in this limit the microscopic meson current, Eq. (31), can be cast into the same form as the phenomenological meson current, Eq. (16), i.e.

$$J_\nu^{\mathrm{micro}}(\vec{k}_M'; \vec{k}_M) \stackrel{|\vec{k}_M| \to \infty}{\longrightarrow} (Q_q + Q_{\bar{q}})\,(k_M' + k_M)_\nu \, F(Q^2)\,. \tag{35}$$

This allows us to extract the electromagnetic form factor directly from the limiting expression of $J_\nu^{\mathrm{micro}}(\vec{k}_M'; \vec{k}_M)$ without making use of Eq. (33). The final result is

$$F(Q^2) = \lim_{|\vec{k}_M| \to \infty} f(\Delta m = Q, |\vec{k}_M|) = \int \mathrm{d}^3 \tilde{k}_q' \sqrt{\frac{m_{q\bar{q}}}{m_{q\bar{q}}'}}\, \mathcal{S}\, u_{n0}^*\,(|\vec{\tilde{k}}_q'|)\, Y_{00}^*(\hat{\tilde{k}}_q')\, u_{n0}\,(|\vec{\tilde{k}}_q|)\, Y_{00}(\hat{\tilde{k}}_q)\,. \tag{36}$$

The spin-rotation factor $\mathcal{S}$ is the $|\vec{k}_M| \to \infty$ limit of the trace of the Wigner D function occurring in Eq. (31). In order to find explicit expressions for $\tilde{\vec{k}}_q$, $m_{q\bar{q}}$ and $\mathcal{S}$ in terms of the integration variable $\tilde{\vec{k}}'_q$ and the momentum transfer Q we have to specify our kinematics. For convenience we choose the (1,3)-plane as the scattering plane and the 3-momenta such that

$$\vec{k}_M = \begin{pmatrix} -\frac{Q}{2} \\ 0 \\ \sqrt{\vec{k}_M^2 - \frac{Q^2}{4}} \end{pmatrix} , \qquad \vec{q} = \begin{pmatrix} Q \\ 0 \\ 0 \end{pmatrix} , \qquad \vec{k}'_M = \vec{k}_M + \vec{q} \, . \tag{37}$$

It should be noted that Q is restricted by $Q < 2|\vec{k}_M|$. With this choice of kinematics we find [17]

$$\tilde{\vec{k}}_q \overset{|\vec{k}_M| \to \infty}{\longrightarrow} \begin{pmatrix} \tilde{k}_q^{1\prime} + \left(\frac{\tilde{k}_q^{3\prime}}{m'_{q\bar{q}}} - \frac{1}{2} \right) Q \\ \tilde{k}_q^{2\prime} \\ \tilde{k}_q^{3\prime} \frac{m_{q\bar{q}}}{m'_{q\bar{q}}} \end{pmatrix} , \tag{38}$$

$$m_{q\bar{q}} \overset{|\vec{k}_M| \to \infty}{\longrightarrow} \sqrt{m'^2_{q\bar{q}} - \frac{4\tilde{k}_q^{1\prime} m'_{q\bar{q}}}{2\tilde{k}_q^{3\prime} + m'_{q\bar{q}}} Q + \frac{\left(m'_{q\bar{q}} - 2\tilde{k}_q^{3\prime} \right)}{2\tilde{k}_q^{3\prime} + m'_{q\bar{q}}} Q^2} \, , \tag{39}$$

and

$$\mathcal{S} = \frac{m'_{q\bar{q}}}{m_{q\bar{q}}} - \frac{2\tilde{k}_q^{\prime 1} \, Q}{m_{q\bar{q}} (m'_{q\bar{q}} + 2\tilde{k}_q^{\prime 3})} \, . \tag{40}$$

$F(Q^2)$ is thus independent of the reference frame. The integrand on the right-hand side of Eq. (36) depends only on the momentum transfer Q and the internal (anti)quark momentum $\tilde{\vec{k}}'$, which is integrated over. Details of the confinement dynamics enter solely via the form of the bound-state wave function $u_{n0} \left(|\tilde{\vec{k}}_q| \right) Y_{00}(\hat{\tilde{\vec{k}}}_q)$ and not via the mass of the bound $q\bar{q}$ cluster. What we have achieved with Eq. (36) is an impulse approximation to the electromagnetic meson form factor. In the limit $|\vec{k}_M| \to \infty$ the whole photon momentum is transferred to one of the constituents (since $q^\mu \overset{|\vec{k}_M| \to \infty}{\longrightarrow} \underline{q}^\mu$), whereas the other one acts as a spectator. Furthermore it is important to note that the electromagnetic meson current (expressed in terms of physical particle momenta) $J_\nu^{\text{micro}}(\vec{p}'_M; \vec{p}_M)$ acquires the correct continuity, covariance and cluster-separability properties in the limit $|\vec{k}_M| \to \infty$, as can directly be seen from boosting Eq. (35) by $B_c(v)$. We can also turn this around and say that we have found a reference frame for the $\gamma^* M \to M$ subprocess in which a one-body constituent current already provides the correct continuity, covariance and cluster-separability properties for the electromagnetic meson current.

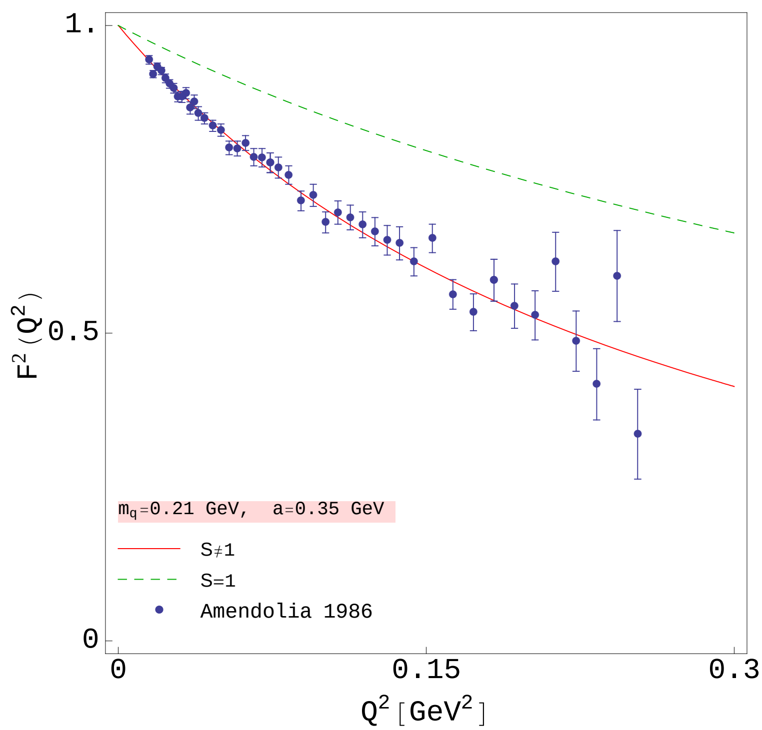

FIG. 3: Q^2 dependence of the pion form factor in the low-Q^2 region, as evaluated by means of Eq. (36) with (solid) and without (dashed) spin-rotation factor $\mathcal{S}$. Data are taken from Ref. [20]

Numerical results obtained with our final expression for the electromagnetic pion form factor, Eq. (36), and the simple harmonic-oscillator wave function introduced above are depicted in Figs. 3 and 4 along with experimental data. The one-body constituent current, which we end up with, together with a simple two-parameter harmonic-oscillator wave function is seen to provide a reasonable fit to the pion electromagnetic form factor. Also shown is the role of the spin-rotation factor $\mathcal{S}$. The quark spin obviously has a substantial effect on the electromagnetic form factor over nearly the whole momentum-transfer range and thus cannot be neglected. As will become clear in the following all these findings are not too surprising. They coincide with corresponding statements made in Ref. [19] (from where we have taken the wave function parametrization).

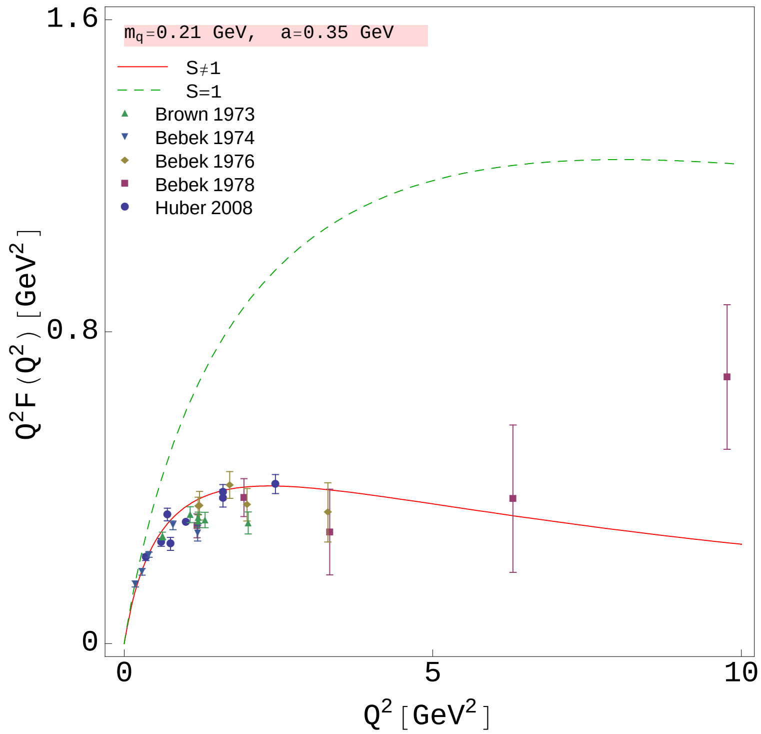

FIG. 4: Q^2 dependence of the pion form factor scaled by Q^2, as evaluated by means of Eq. (36) with (solid) and without (dashed) spin-rotation factor $\mathcal{S}$. Data are taken from Refs. [21, 22, 23, 24, 25]

IV. COMPARISON WITH OTHER APPROACHES

A. Comparison with front-form results

The fact that we extract the electromagnetic meson form factor in the limit $|\vec{k}_M| \to \infty$ means that the $\gamma^* M \to M$ subprocess is considered in the infinite momentum frame of the meson. This offers the possibility of making a direct comparison with form factor analyses using front form of relativistic dynamics. The $|\vec{k}_M| \to \infty$ limit of the kinematics chosen in Eq. (37) implies in particular that we work in a reference frame in which the plus component of the 4-momentum transfer $q^\mu = (k'_M - k_M)^\mu$ vanishes. $q^+ = 0$ frames are also popular for form-factor studies in front form [4, 19, 26, 27]. One reason is that the impulse approximation can be formulated consistently in any $q^+ = 0$ frame (for the plus component of the current operator) [4]. The other reason is that so called "Z-graphs" are suppressed in such frames [27].

Our point form calculation can be related to front form results by an appropriate change

of variables. To show this we define the longitudinal momentum fractions ξ and ξ' as

$$\xi^{(\prime)} - \frac{1}{2} = \frac{\tilde{k}_q^{3(\prime)}}{m_{q\bar{q}}^{(\prime)}} \tag{41}$$

and introduce the short-hand notation

$$\kappa_\perp^{(\prime)} = \begin{pmatrix} \tilde{k}_q^{1(\prime)} \\ \tilde{k}_q^{2(\prime)} \end{pmatrix} \tag{42}$$

for the intrinsic transverse momentum of the active incoming (outgoing) quark. From Eq. (38) we infer that

$$\xi = \xi' \quad \text{and} \quad \kappa'_\perp = \kappa_\perp + (1-\xi)\,Q_\perp \quad \text{with} \quad Q_\perp = \begin{pmatrix} Q \\ 0 \end{pmatrix} . \tag{43}$$

The invariant mass of the incoming (outgoing) $q\bar{q}$-system can then be written as

$$m_{q\bar{q}}^{(\prime)} = \sqrt{\frac{m_q^2 + \kappa_\perp^{(\prime)2}}{\xi(1-\xi)}} . \tag{44}$$

With these relations the Jacobian for the variable transformation $(\tilde{k}_q^{1\prime}, \tilde{k}_q^{2\prime}, \tilde{k}_q^{3\prime}) \rightarrow (\xi, \kappa_\perp^1, \kappa_\perp^2)$ becomes

$$\frac{\partial(\tilde{k}_q^{1\prime}, \tilde{k}_q^{2\prime}, \tilde{k}_q^{3\prime})}{\partial(\xi, \kappa_\perp^1, \kappa_\perp^2)} = \frac{m'_{q\bar{q}}}{4\xi(1-\xi)} \tag{45}$$

and the integral for the electromagnetic form factor takes on the form

$$F(Q^2) = \frac{1}{4\pi} \int_0^1 \mathrm{d}\xi \int_{\mathbb{R}^2} \mathrm{d}^2\kappa_\perp \frac{\sqrt{m_{q\bar{q}}\, m'_{q\bar{q}}}}{4\xi(1-\xi)} \,\mathcal{M}\, u_{n0}^*\,(|\vec{\tilde{k}}'_q|)\, u_{n0}\,(|\vec{\tilde{k}}_q|) . \tag{46}$$

The argument of the wave functions is easily expressed in terms of ξ and $\kappa_\perp$ if one uses $\vec{\tilde{k}}_q^{(\prime)2} = m_q^2 + m_{q\bar{q}}^{(\prime)\,2}/4$ and Eq. (44). By the change of variables the spin-rotation factor $\mathcal{S}$ (cf. Eq. (40)) goes over into the Melosh-rotation factor

$$\mathcal{M} = \frac{m_{q\bar{q}}}{m'_{q\bar{q}}} \left(1 + \frac{(1-\xi)\,(Q_\perp \cdot \kappa_\perp)}{m_q^2 + \kappa_\perp^2} \right) . \tag{47}$$

$\mathcal{S}$ and $\mathcal{M}$ describe the effect of the quark spin onto the electromagnetic form factor in point form and in front form, respectively. Equations (46) and (47) are identical to the corresponding formulae in Refs. [19, 27]. This is a remarkable result. Starting from two different forms of relativistic dynamics and applying completely different procedures to identify the electromagnetic meson form factor the outcome is the same. It means that relativity is treated

in an equivalent way and the physical ingredients are alike. Since the infinite-momentum frame we use is just a particular $q^+ = 0$ frame, Z-graph contributions to the electromagnetic meson form factor are also suppressed in our point-form approach. This is a welcome feature, because Z-graphs can play a significant role in $q^+ \neq 0$ frames [27] and one should have control on them when form-factor predictions are compared with experiment.

B. Comparison with the point-form spectator model

Relativistic point-form quantum mechanics has also been applied in Ref. [13, 14] to calculate electroweak baryon form factors within a constituent quark model. The strategy for the extraction of hadron form factors, however, differs from the one in the present paper. The Bakamjian-Thomas type framework which we apply to the full electron-hadron system in order to derive the electromagnetic hadron current is only used to obtain the bound-state wave function of the hadron. This wave function is then plugged into an ansatz for the electromagnetic hadron current. The ansatz is constrained by the requirements that the hadron current should be conserved and should have the correct properties under space-time translations and Lorentz transformations. It is shown that these constraints can be satisfied by a spectator current if not all of the photon momentum is transferred to the struck quark. The momentum transfer to the active quark $\tilde{Q}$ is uniquely determined by total 4-momentum conservation for the $\gamma^* H \to H$ subprocess and by the spectator conditions. An ambiguity in defining such a spectator current, however, enters through a normalization factor $\mathcal{N}$ which has to be introduced in order to recover the hadron charge from the electric form factor in the limit $Q^2 \to 0$ [28]. Since both quantities, $\tilde{Q}$ and $\mathcal{N}$ depend effectively on all quark momenta and not only on those of the active ones, the model current constructed in this way cannot be considered as a pure one-body current [29]. It has therefore been termed "point-form spectator model" (PFSM) to distinguish it from the usual impulse approximation.

Another characteristic feature of PFSM form factors is that they are determined not only by the hadron bound-state wave function, but also exhibit an explicit dependence on the mass of the bound state. Within the PFSM the eigenvalue spectrum of the mass operator is thus directly connected with the electromagnetic structure of its eigenstates. This makes it somewhat delicate to compare our form-factor results for the simple harmonic-oscillator confinement potential with corresponding PFSM predictions. Whereas the wave function

is solely determined by the oscillator parameter a (cf. Eq. (34)), another free constant V_0 can be added to the confinement potential to shift the spectrum. Unlike our results, which do not depend on V_0, the PFSM predictions exhibit a strong dependence on V_0. Taking $a = 0.35$ GeV (as above) and $V_0 < 0$, such that if the harmonic oscillator ground state were to coincide with the physical mass of the pion, the fall-off of the form factor would be unreasonably strong. With $V_0 = 0$, on the other hand, the pion ground-state mass would be larger than 1 GeV and the fall-off of the form factor would be much too slow. We have therefore tried to take another set of parameters for the harmonic-oscillator confinement potential which is fixed through the vector-meson spectrum [18]. With this set of parameters, i.e. $a = 0.312$ GeV, $V_0 = -1.04$ GeV^2 and $m_q = 0.34$ GeV, the masses of the vector meson ground states and first excited states are reasonably well reproduced. Applying them to the π meson and its excitations we observe that the first and second radial excitations are about 10% too high as compared with experiment and the π ground state has a mass of 0.77 GeV. These are reasonable values for a pure central confining potential in view of the fact that spin-spin forces from an additional hyperfine interaction can bring them close to the experimental masses [30]. But for the purpose of a qualitative comparison of our coupled-channel formalism with the PFSM we just stay with the simple harmonic-oscillator q-$\bar{q}$ potential. If it is parameterized as in Ref. [18] the predictions for the electromagnetic π form factor obtained by means of Eq. (36) and with the PFSM become comparable at small momentum transfers (cf. Fig.5). Above $Q^2 \approx 1$ GeV^2, however, significant differences can be observed. These differences resemble the situation for the electromagnetic nucleon form factors. In the latter case the stronger fall-off produced by the PFSM is a welcome feature which brings the theoretical predictions from constituent quark models close to experiments [29]. For the usual front-form spectator current in the $q^+ = 0$ frame agreement with experiment is achieved only by introducing electromagnetic form factors for the constituent quarks [31]. It remains to be seen whether the situation for the electromagnetic π form factor could also change in favor of the PFSM if a more sophisticated q-$\bar{q}$ potential is employed which reproduces the mass of the π meson and its lowest excitations sufficiently accurately.

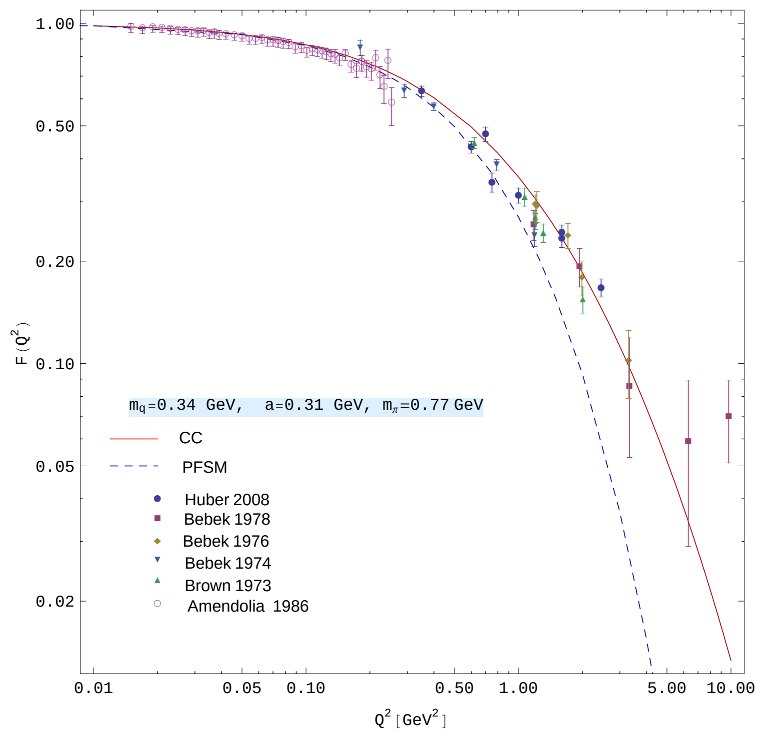

FIG. 5: The electromagnetic pion form factor $F\left(Q^2\right)$ as evaluated by means of Eq. (36) (solid line) in comparison with the outcome of the point-form spectator model (dotted line). Data as in Figs. 3 and 4.

V. SUMMARY AND OUTLOOK

We have analyzed the electromagnetic structure of pseudoscalar mesons by utilizing the point form of relativistic quantum mechanics in connection with a velocity-state representation. In this formulation the scattering of an electron by a confined quark-antiquark pair becomes a two-channel problem for a Bakamjian-Thomas type mass operator. With the second channel, the $eq\bar{q}\gamma$ channel, the dynamics of the exchanged photon is explicitly taken into account. Confinement is treated via an instantaneous potential. The emission and absorption of a photon by an electron or (anti)quark is described by a vertex interaction which has the Lorentz structure of the field theoretical vertex, but conserves the 4-velocity of the whole $eq\bar{q}(\gamma)$ system. By construction this approach is Poincaré invariant. After reduction of the eigenvalue problem for the coupled-channel mass operator to a non-linear eigenvalue equation for the $eq\bar{q}$ channel, the matrix elements of the electromagnetic meson current can be read off from the one-photon-exchange optical potential. As expected the optical potential

is a contraction of the point-like electron current with a meson current times the covariant photon propagator. The resulting meson current transforms as a 4-vector and satisfies current conservation. The extracted meson form factor, however, depends not only on Q^2, i.e. the 4-momentum transfer squared, but also on Mandelstam s, i.e. the total invariant mass squared of the electron-meson system. This finding hints at a violation of cluster separability, since the meson current is also influenced by the presence of the electron. The violation of cluster separability is a consequence of the approximation that the total velocity of the electron-meson system is conserved throughout the electromagnetic scattering process. On the other hand, it is only because of this approximation that we have been able to formulate electron-meson scattering as a simple eigenvalue problem for a Bakamjian-Thomas type mass operator. A possible way to recover cluster separability is the introduction of so called "packing operators" [4, 5, 6], i.e. appropriate unitary transformations which restore cluster separability. We have not made use of such packing operators, but have rather exploited the observation that the s-dependence of the meson form factor vanishes rapidly with increasing s, indicating that cluster-separability-violating effects become negligible if the meson momentum is large enough. It has been shown analytically that in the limit of infinitely large meson momentum the current goes over into a product of the usual point-like meson current times an integral with the integrand depending only on internal variables (which are integrated and summed over) and on the momentum transfer Q. The limit of the current is a one-body current. The physical meaning of letting the meson momentum go to infinity is that the $\gamma^* M \to M$ subprocess is considered in the infinite momentum frame of the meson. We have thus succeeded in deriving a conserved electromagnetic current for a composite pseudoscalar meson which acquires the correct covariance properties under Poincaré transformations if the meson momentum is sufficiently large. It should be emphasized that this is no restriction on the (space-like) momentum transfer Q. Finally we have been able to prove that our analytical formula for the form factor is equivalent to the usual front-form expression that results from a spectator current in the $q^+ = 0$ frame.

Apart from confirming that the front- and point-form of relativistic dynamics lead to equivalent results for the electromagnetic form factor of a pseudoscalar meson, it might seem that there is nothing new in these results. What makes our approach interesting, however, is that it can immediately be generalized in various directions. It is quite obvious how to proceed for (space-like) form factors of arbitrary few-body systems which are bound by

an instantaneous potential. Even more, one could account for dynamical particle-exchange interactions by adding additional channels. In this case the current of the bound few-body system would not be a simple one-body current, but would also contain many-body contributions. In general, the construction of such many-body currents is a highly nontrivial task [32]. The advantage of our approach is that the current is uniquely determined by the interaction dynamics which is responsible for the binding and can be read off directly from the one-photon exchange optical potential in the electron-cluster channel. The only problem with this procedure is connected with cluster separability. For the simple example treated in this paper this problem has been overcome by going to the infinite-momentum frame of the cluster. It remains to be seen whether the same strategy also works in the general case, or whether one has to apply packing operators to ensure cluster separability right from the beginning.

In principle, one could also think of applying our coupled-channel formalism to the calculation of time-like hadron form factors. One has to study the process $e^- e^+ \rightarrow \gamma^* \rightarrow H\bar{H}$. In this case, however, the situation becomes much more complicated. It is not enough to know the bound-state dynamics of the hadron H. It is also necessary to specify the (strong) interaction mechanism that produces those hadronic constituents which do not directly couple to the photon. In the time-like case the argument of the form factor is Mandelstam s. It is therefore not possible to go into the infinite momentum frame in order to get rid of problems with cluster separability, which most likely will show up as additional angular dependence of the form factors. A possible way out could again be the application of packing operators.

To summarize, we have presented a relativistic formalism which makes it possible to derive the electromagnetic current and form factors of a bound few-body system consistent with the binding forces. In this paper the formalism has been worked out in detail for a quark-antiquark pair with the quantum numbers of a pion which is confined via an instantaneous potential. The formalism, however, is much more general and applications to other systems with more complicated bound-state dynamics will be the subject of further investigations.

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[35] More recently two-photon-exchange effects have also been studied in connection with electron-nucleon scattering [33, 34].

[36] Note that the velocity-state representation is associated with center-of-mass kinematics, c.f. Eq. (4).

[37] Note that $\vec{q} = \vec{k}_\gamma$, but $q^0 \neq k^0_\gamma = \omega_{k_\gamma} = |\vec{k}_\gamma|$.